

XXXVI. *On the Precession of the Equinoxes.* By the Rev.
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1. **T**HE true cause of the precession of the equinoctial points was first assigned by Sir ISAAC NEWTON; but it is confessed, that he has fallen into an error in his investigation of the effect. Without, however, entering into any enquiry relative to the circumstances in which he has erred, I propose to shew how we may obtain a true solution from his own principles, by means of which alone the whole calculation may be rendered extremely simple and evident: and although very satisfactory solutions have been already given, yet the importance of the problem will sufficiently apologise for offering any thing further upon the subject that may at all tend to elucidate it.

2. Let S (Tab. XV. fig. 1.) be the sun, ABDC the earth, T its center, EQ the equator, P, p , the poles; draw CTB perpendicular to SAD, and join SE, which produce to meet CB in K. Call the radius ET unity, and let the force of the sun on a particle at T be $\frac{1}{ST^2}$, then the force on a particle at E = $\frac{1}{SE^2}$; hence, if we resolve this latter force into two others, one in the direction ET, and the other in the direction parallel to TS, we have $SE : ST :: \frac{1}{SE^2} : \text{the force in the direction parallel to}$

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to $TS = \frac{ST}{SE^3} = \frac{ST}{ST-EK^3} = \frac{1}{ST^2} + \frac{3EK}{ST^3}$, omitting the other terms of the series on account of their smallness. Hence the force with which a particle at E is drawn from CB is equal to $\frac{3EK}{ST^3}$; consequently the effect of this force in a direction perpendicular to ET will be $\frac{3EK \times KT}{ST^3}$; hence this force : the force of the sun on a particle at T :: $\frac{3EK \times KT}{ST^3} : \frac{1}{ST^2} :: 3EK \times KT : ST$. Now if P = the periodic time of the earth, p = the periodic time of a body revolving at the earth's surface; then the force of the earth to the sun : the force of the body to the earth, or the force of gravity, :: $\frac{ST}{P^2} : \frac{1}{p^2}$; and hence the force of the sun on a particle at E perpendicular to ET : the force of gravity :: $\frac{3EK \times KT \times p^2}{P^2} : 1$.

3. Let v be the center of gyration, and put M = the quantity of matter in the earth : then the effect of the inertia of M placed at v , to oppose the communication of motion, is the same as the effect of the inertia of the earth; and hence, by the property of that center, $ET^2 : Tv^2 (= \frac{2}{3} ET^2) :: M : \frac{2}{3} M$, which is the quantity of matter to be placed at E to have the same effect.

4. Put m = the excess of the quantity of matter in the earth above that of its inscribed sphere. Now by Sir ISAAC NEWTON's two first lemmas, it appears, that the action of the sun upon the shell m of matter, to generate an angular velocity about an axis perpendicular to CABD, is just the same as it would be to generate an angular velocity in a quantity of matter equal to $\frac{1}{3} m$ placed at E. Let us therefore suppose the sun's attraction, perpendicular to ET, to be exerted upon a
quantity

quantity of matter at E equal to $\frac{1}{5} m$, and at the same time to have a quantity of matter to move equal to $\frac{2}{5} M$, and then from this and art. 3. it appears, that the effect will be the same as the accelerative force of the sun to turn about the earth. Hence that accelerative force is, from art. 2. equal to $\frac{3EK \times KT \times p^2 \times \frac{1}{5} m}{\frac{2}{5} M \times P^2} = \frac{3EK \times KT \times p^2 \times m}{2M \times P^2}$, gravity being unity. Now, if $TE : TP :: 1 : 1 - r$, then $M : M - m :: 1 : 1 - 2r$, therefore, $M : m :: 1 : 2r$, hence $\frac{m}{2M} = r$, consequently the accelerative force = $\frac{3EK \times KT \times p^2 \times r}{P^2}$.

5. Let \dot{z} = the arc described by a point of the equator about its axis in an indefinitely small given time, which may therefore represent its velocity; and let $a\dot{z}$ represent the arc described in the same time by a body revolving about the earth at its surface; then $\frac{a^2 \dot{z}^2}{2}$ = the sagitta of the arc described by the body in the same time, and consequently $a^2 \dot{z}^2$ = the velocity generated by gravity whilst a point of the equator describes \dot{z} . Hence, by art. 4. we have $1 : \frac{3EK \times KT \times p^2 \times r}{P^2} :: a^2 \dot{z}^2 : \frac{3EK \times KT \times p^2 \times r \times a^2 \dot{z}^2}{P^2}$ the velocity of the point E of the equator generated by the action of the sun, whilst the equator describes \dot{z} about its axis; consequently the ratio of these velocities is as $\frac{3EK \times KT \times p^2 \times a^2 \dot{z}^2}{P^2} : 1$.

6. Let y be an arc described by the sun in the ecliptic to a radius equal to unity, whilst a point of the equator describes \dot{z} about its axis; then (as ap = the time of the earth's rotation, and the arcs described in equal times to equal radii are inversely as the periodic times) $\frac{1}{P} : \frac{1}{ap} :: y : \dot{z} = \frac{Pj}{ap}$; hence, if v and w be put
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for the sine and cosine of the sun's declination, the ratio of the velocities in the last article becomes $\frac{3aprvw\dot{y}}{P} : 1$.

7. Hence if TSL (fig. 2.) be the ecliptic to the radius unity, P the plane of the sun, SER the equator, PE the sun's declination, and we take $Ec : cd$ (cd being perpendicular to Ec) $:: 1 : \frac{3aprvw\dot{y}}{P}$, and through d , E, describe the great circle TEM, then will ST be the precession of the equinox during the time the sun describes y in the ecliptic. Now Ed (or Ec , as the angle at E is indefinitely small) $: dc :: \text{rad.} = 1 : \text{fine angle } E = \frac{3aprvw\dot{y}}{P}$; hence (if SV be drawn perpendicular to TE) $1 : \text{fine } SE :: \frac{3aprvw\dot{y}}{P} : SV = \frac{3aprvw \times \text{fin. } SE \times \dot{y}}{P}$; therefore, $\text{fin. } STV \text{ or } ESP : 1 :: SV : ST = \frac{3aprvw \times \text{fin. } SE \times \dot{y}}{P \times \text{fin. } ESP}$.

8. Now $\frac{v}{\text{fin. } ESP} = \text{fin. } SP$, and $w = \frac{\text{cof. } SP}{\text{cof. } ES}$, hence $\frac{vw}{\text{fin. } ESP} = \frac{\text{fin. } SP \times \text{cof. } SP}{\text{cof. } ES}$; but $\frac{\text{cof. } ESP}{\text{tan. } ES \times \text{cot. } SP} = 1$, hence $\frac{vw}{\text{fin. } ESP} = \frac{\text{fin. } SP \times \text{cof. } SP \times \text{cof. } ESP}{\text{cof. } ES \times \text{tan. } ES \times \text{cot. } SP} = \frac{\text{fin. } SP^2 \times \text{cof. } ESP}{\text{fin. } ES}$; consequently,

$$ST = \frac{3apr \times \text{fin. } SP^2 \times \text{cof. } ESP \times \dot{y}}{P} = (\text{if } x = \text{fin. } SP)$$

$\frac{3apr \times \text{cof. } ESP \times x^2 \dot{x}}{P \times \sqrt{1-x^2}}$, whose fluent, when $x = 1$, is $\frac{3apr \times \text{cof. } ESP \times y}{2P}$

(y being now = to a quadrant) the arc of precession whilst the sun describes 90° of the ecliptic; and to find the degrees say,

$$4y : 360^\circ :: \frac{3apr \times \text{cof. } ESP \times y}{2P} : 360^\circ \times \frac{3apr \times \text{cof. } ESP}{8P}$$

the precession in a year $= 360^\circ \times \frac{3apr \times \text{cof. } ESP}{2P} = 21'' 6'''$. This

would be the precession of the equinox arising from the attraction of the sun, if the earth were of an uniform density, and the ratio of

Fig. 1.

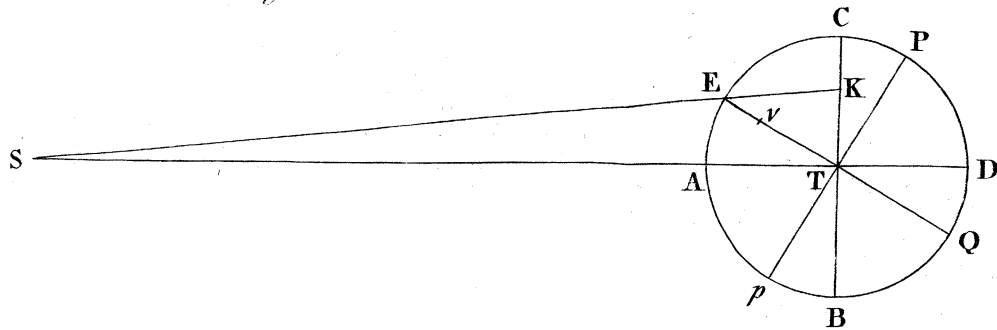
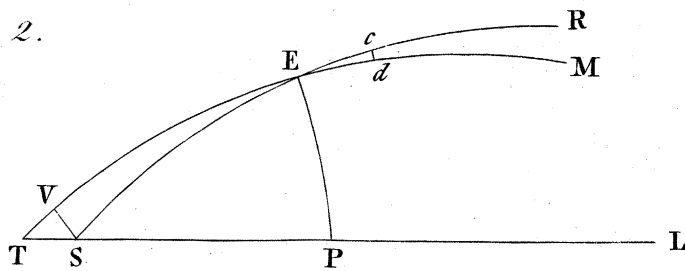


Fig. 2.



of the diameters as 229 : 230 ; but if the greatest nutation of the earth's axis be rightly ascertained, the precession is only about $14''\frac{2}{3}$; which difference between the theory and what is deduced from observation must arise either from the fluidity of the earth's surface, an increase of density towards the center, or the ratio of the diameters being different from what is here assumed, or probably from all the causes conjointly. But as the best observations must be liable to some small degree of inaccuracy, and an error of one or two seconds in the nutation will, in this case, make a very considerable alteration in the conclusion, the estimation of the precession arising from the action of the sun seems to be subject to a very considerable degree of uncertainty.

